Advances in Bayesian real-time model updating

贝叶斯实时模型更新进展

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Overview

1. Introduction
   - Why Bayesian?

2. Real-time model updating
   2.1. Conventional approach and its problem
   2.2. Outlier cleansing
   2.3. Noise parameters updating

3. Example

4. Concluding remarks
1. Introduction

Real-time model updating
- Outliers detection and cleansing
- Stationary/nonstationary circumstances
Why Bayesian?

- Uncertainty Quantification
  - not only the optimal point
  - indication of unidentifiability

- More objective consideration of the objective function
  - spectrum
  - correlation function

- Weighting for subsequent analysis

- Model Class Selection

- Misunderstanding of prior distribution
2. Real-time Model Updating

Proposed method

Stable Robust Extended Kalman Filter (SREKF)

Objectives

Real-time model updating:
- for linear/nonlinear systems
- under stationary/nonstationary response
- with outlier detection and cleansing
Conventional Approach: Extended Kalman filter

State-space equation:

\[ \mathbf{y}_{k+1} = \mathbf{A}_n \mathbf{y}_k + \mathbf{B}_n \mathbf{f}_k + \tilde{\mathbf{a}}_k \]

Measurement:

\[ \mathbf{z}_{k+1} = \mathbf{C} \mathbf{y}_{k+1} + D \bar{\mathbf{f}}_{k+1} + \mathbf{\varepsilon}_{k+1} \]

Predicted state:

\[ \hat{\mathbf{y}}_{k+1|k} = \mathbf{A}_k \hat{\mathbf{y}}_{k|k} + \mathbf{B}_k \bar{\mathbf{f}}_k + \tilde{\mathbf{g}}_k \]

\[ \hat{\Sigma}_{\mathbf{y},k+1|k} = \mathbf{A}_k \hat{\Sigma}_{\mathbf{y},k|k} \mathbf{A}_k^T + \mathbf{B}_k \hat{\Sigma}_{\mathbf{f}} \mathbf{B}_k^T \]

Updated state:

\[ \mathbf{K}_{k+1} = \hat{\Sigma}_{\mathbf{y},k+1|k} \mathbf{C}^T \left( \mathbf{C} \hat{\Sigma}_{\mathbf{y},k+1|k} \mathbf{C}^T + \mathbf{\Sigma}_{\mathbf{\varepsilon}} \right)^{-1} \]

\[ \hat{\mathbf{y}}_{k+1|k+1} = \hat{\mathbf{y}}_{k+1|k} + \mathbf{K}_{k+1} (\mathbf{z}_{k+1} - \mathbf{C} \hat{\mathbf{y}}_{k+1|k} - D \bar{\mathbf{f}}_{k+1}) \]

\[ \hat{\Sigma}_{\mathbf{y},k+1|k+1} = \mu \left( \mathbf{I}_{N_y} - \mathbf{K}_{k+1} \mathbf{C} \right) \hat{\Sigma}_{\mathbf{y},k+1|k} \]

Outlier contaminated measurement

Ad hoc assumption on the noise covariance matrices
Formulation of the Proposed method

Initial $\hat{y}_{0|0}$, $\hat{\Sigma}_{y,0|0}$, $\hat{\theta}_{0|0}$ and $\hat{\Sigma}_{\theta,0|0}$

Obtain $A_k$, $B_k$ and $\tilde{g}_k$

**Predict:** Compute $\hat{y}_{k+1|k}$ and $\hat{\Sigma}_{y,k+1|k}$

Measurement $z_{k+1}$

Outlier cleansing

Noise parameters updating

**Filter:** Compute $\hat{y}_{k+1|k+1}$ and $\hat{\Sigma}_{y,k+1|k+1}$

$k = k + 1$
Gymnastics Scoring System

- Why do we trim the highest and lowest score?

  Score is dominated by this referee (outlier) → Trimming → Robust estimation of the performance
Formulation: Outlier Cleansing

Existing criterion

$$\frac{\varepsilon}{\sigma} > 2.5$$
Formulation: Outlier Cleansing

Normalized prediction error of the measurements of the $s^{th}$ channel:

$$
\varepsilon_{k+1}^{(s)} = \left( z_{k+1}^{(s)} - z_{k+1|k}^{(s)} \right) / \sigma_{\varepsilon,k+1}^{(s)}
$$

Probability of a data point falling outside the interval $\left( -\varepsilon_{k+1}^{(s)}, \varepsilon_{k+1}^{(s)} \right)$:

$$
Q_{k+1}^{(s)} = 2 \Phi \left( -\varepsilon_{k+1}^{(s)} \right)
$$
Formulation: Outlier Cleansing

Probability of abnormality depending only on the $N_w$-time window:

$$P_A(z_{k+1}^{(s)}) \approx \begin{cases} 
\sum_{\gamma=0}^{\Gamma_{k+1}^{(s)^*}} C(n_k^{(s)} + 1, \gamma) \cdot (Q_{k+1}^{(s)})^\gamma \cdot (1 - Q_{k+1}^{(s)})^{n_k^{(s)}+1-\gamma}, & \text{if } n_k^{(s)} < N_w \\
\sum_{\gamma=0}^{\Gamma_{k+1}^{(s)^*}} C(N_w + 1, \gamma) \cdot (Q_{k+1}^{(s)})^\gamma \cdot (1 - Q_{k+1}^{(s)})^{N_w+1-\gamma}, & \text{if } n_k^{(s)} \geq N_w 
\end{cases}$$

$$\begin{cases} 
P_A(z_{k+1}^{(s)}) < 0.5, & Z_{k+1}^{(s)} \text{ is classified as a regular point} \\
\text{otherwise}, & Z_{k+1}^{(s)} \text{ is classified as an outlier to be removed}
\end{cases}$$
Formulation: Outlier Cleansing

\[ z_{k+1} = \begin{bmatrix} z^{(1)} \\ z^{(2)} \\ z^{(3)} \end{bmatrix}_{k+1} \]

\[ z_{k+1} = \begin{bmatrix} z^{(1)} \\ z^{(2)} \\ z^{(3)} \end{bmatrix}_{k+1} \]
Formulation: Outlier Cleansing

\[ z_{k+1} = \begin{bmatrix} z^{(1)}_{k+1} \\ z^{(2)}_{k+1} \\ z^{(3)}_{k+1} \end{bmatrix} \]

\[ \hat{y}_{k+1|k+1} = \hat{y}_{k+1|k} \]

\[ \hat{\Sigma}_{y,k+1|k+1} = \hat{\Sigma}_{y,k+1|k} \]
Formulation: Noise Parameters Updating

Bayes’ theorem:

\[
p(\theta_{k+1}|D_{k+1}) = \frac{p(\theta_{k+1}|D_k)p(z_{k+1}|\theta_{k+1}, D_k)}{p(z_{k+1}|D_k)}
\]

Noise parameter vector: \( \theta_{k+1} = [\theta_{f,k}^T, \theta_{\varepsilon,k+1}^T]^T \)

Objective function: \( J = -\ln p(\theta_{k+1}|D_{k+1}) \)

Updated noise parameter vector:

\[
\hat{\theta}_{k+1|k+1} = \arg\min_{\theta_{k+1}} J(\theta_{k+1})
\]

Covariance matrix of \( \hat{\theta}_{k+1|k+1} \):

\[
\hat{\Sigma}_{\theta,k+1|k+1} = \left[ H_J(\hat{\theta}_{k+1|k+1}) \right]^{-1} = \left[ \nabla J(\theta_{k+1}) \nabla^T \right]_{\theta_{k+1} = \hat{\theta}_{k+1|k+1}}^{-1}
\]
Formulation: Noise Parameters Updating

Preparatory phase: *Half – or – double algorithm*

\[ \hat{\theta}_{k+1|k+1} = \arg\min_{\theta_{k+1} \in \Theta_{k+1}} J(\theta_{k+1}) \]
Formulation: Noise Parameters Updating

Preparatory phase: \textit{Half – or – double algorithm}

\[ \hat{\theta}_{k+1|k+1} = \text{arg} \min_{\theta_{k+1} \in \Theta_{k+1}} J(\theta_{k+1}) \]

Termination criteria:

\begin{itemize}
  \item \( t \geq 10 \, T_n \)
  \item \( \hat{\theta}_{k+1|k+1} = \hat{\theta}_{m|m}, m = k, k - 1, ..., k - 9 \)
\end{itemize}

Operation phase: Gradient method

\[ \hat{\theta}_{k+1|k+1} = \hat{\theta}_{k|k} - [H_J(\hat{\theta}_{k|k})]^{-1} \nabla J(\hat{\theta}_{k|k}) \]
3. Example: Real-time Model Updating

Modulated function of excitation and measurement noise:

\[ A_f(t) = \begin{cases} 
1, & 0 \leq t \leq T_A \\
1 + \frac{t - T_A}{T_B} \exp \left(1 - \frac{t - T_A}{T_B}\right), & T_A < t \leq T 
\end{cases} \]

Damage:

- 1/F: -10% stiffness at \( t = 100 \) sec
- 3/F: -10% stiffness at \( t = 150 \) sec

Measurement:

- Sensor locations: 1, 3, 5, 7, 10, 15 and 20/F
- Outlier occurrence rate: 1%
## Example: Outlier Cleansing Performance

<table>
<thead>
<tr>
<th>Sensor location (Floor)</th>
<th>Masking (%)</th>
<th>Swamping (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.73</td>
</tr>
<tr>
<td>3</td>
<td>0.00</td>
<td>0.46</td>
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<tr>
<td>5</td>
<td>0.00</td>
<td>0.39</td>
</tr>
<tr>
<td>7</td>
<td>0.00</td>
<td>0.44</td>
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<tr>
<td>10</td>
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<td>0.47</td>
</tr>
<tr>
<td>15</td>
<td>0.00</td>
<td>0.42</td>
</tr>
<tr>
<td>20</td>
<td>0.00</td>
<td>0.38</td>
</tr>
</tbody>
</table>
Example: Estimated Noise Parameters
Example: Estimated Noise Parameters
Example: Estimated Model Parameters

**Conventional EKF**

**SREKF**

- $\Psi_K^{(1)}
- $\Psi_K^{(2)}
- $\Psi_K^{(3)}
- $\Psi_K^{(4)}
- $\Psi_K^{(5)}
- $\Psi_K^{(6)}
- $\Psi_K^{(7)}
- $\Psi_K^{(8)}
Example: Comparison of Parametric Estimation Error
4. Concluding Remarks

• SREKF
  - Real-time linear/nonlinear model updating under stationary/nonstationary circumstances with outlier detection and cleaning
    ✔ Online
    ✔ Reliable (stable + robust)
    ✔ Widely applicable
References


Thank You for Your Attention!

Questions and comments are welcome!

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